TABLE 2.1 Elastic Moduli

μ Shear modulus, or rigidity. This is a measure of a material's resistance to shear.

$$\sigma_{ij} = 2\mu\varepsilon_{ij} \Rightarrow \mu = \frac{\sigma_{ij}}{2\varepsilon_{ij}}$$

Note that μ is nonnegative and has units of stress. Typical values are 2×10^{11} dyn/cm^2 or 200 kbar.

k Bulk modulus or incompressibility. k is the material resistance to a change in volume when subject to a load, and it is defined by the ratio of an applied hydrostatic pressure to the induced fractional change in volume:

$$\sigma_{ij} = -P\delta_{ij}, \quad \frac{\Delta V}{V} = \frac{-P}{k} \Rightarrow -P = k\varepsilon_{ii} \Rightarrow \frac{-P}{\varepsilon_{ii}} = \lambda + \frac{2}{3}\mu = k$$

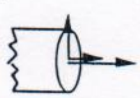
k must be nonnegative, and as a material becomes more rigid, k increases.

λ Lamé's second constant. λ has no simple physical meaning, but it greatly simplifies Hooke's law.

E Young's modulus. E is a measure of the ratio of uniaxial stress to strain in the same direction.

$$\sigma_{11} = E\left(\frac{\Delta L}{L}\right) = E\varepsilon_{11}; \text{ by Hooke's Law, } E = \frac{\mu(3\lambda + 2\mu)}{(\lambda + \mu)}$$

ν Poisson's ratio. ν is the ratio of radial to axial strain when a uniaxial stress is applied ($\sigma_{11} \neq 0$, $\sigma_{22} = \sigma_{33} = 0$).



$$\varepsilon_{22} = \varepsilon_{33}, \quad \nu = \frac{-\varepsilon_{22}}{\varepsilon_{11}} = \frac{\lambda}{2(\lambda + \mu)}$$

Poisson's ratio is dimensionless and has a maximum value of 0.5. This is true for a fluid, when $\mu = 0$ (no shear resistance). The smallest value is 0—infinite shear resistance. Most Earth materials have a Poisson ratio between 0.22 and 0.35.

TABLE 2.2 Relationships between Elastic Moduli

μ	k	λ	E	ν
$\frac{3(k-\lambda)}{2}$	$\lambda + \frac{2\mu}{3}$	$k - \frac{2\mu}{3}$	$\frac{9k\mu}{3k+\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$
$\lambda\left(\frac{1-2\nu}{2\nu}\right)$	$\mu\left[\frac{2(1+\nu)}{3(1-2\nu)}\right]$	$\frac{2\mu\nu}{(1-2\nu)}$	$2\mu(1+\nu)$	$\frac{\lambda}{(3k-\lambda)}$
$3k\left(\frac{1-2\nu}{2+2\nu}\right)$	$\lambda\left(\frac{1+\nu}{3\nu}\right)$	$3k\left(\frac{\nu}{1+\nu}\right)$	$\mu\left(\frac{3\lambda+2\mu}{\lambda+\mu}\right)$	$\frac{3k-2\mu}{2(3k+\mu)}$
$\frac{E}{2(1+\nu)}$	$\frac{E}{3(1-2\nu)}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$3k(1-2\nu)$	$\frac{3k-E}{6k}$