

Special functions such as the *Kronecker delta function* also benefit from indicial notation:

$$\delta_{ij} = \begin{cases} 0 & \text{for } i \neq j \\ 1 & \text{for } i = j \end{cases} \quad i, j = 1, 2, 3 \quad (2.1.4)$$

Throughout this text we assume the *Einstein summation notation*, in which repetition of indices within a term explicitly requires summation on that term. Thus, for a term such as

$$\Delta = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} = \varepsilon_{nn} \quad (2.1.5)$$

the repeated index n implies summation. This holds for repeated indices within any single term:

$$\begin{aligned} x_i y_i &= x_1 y_1 + x_2 y_2 + x_3 y_3 \\ y_{i,i} &= \frac{\partial y_1}{\partial x_1} + \frac{\partial y_2}{\partial x_2} + \frac{\partial y_3}{\partial x_3} \end{aligned} \quad (2.1.6)$$

When a single equation is written with indicial notation, generally a set of equations is implied, as the indices assume all of their permutations. For example, the generalized, linear, isotropic, elastic Hooke's law relating stress (σ_{ij}) and strain (ε_{ij}) terms is given by nine equations:

$$\begin{aligned} \sigma_{11} &= \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu\varepsilon_{11} \\ \sigma_{12} &= 2\mu\varepsilon_{12} \\ \sigma_{13} &= 2\mu\varepsilon_{13} \\ \sigma_{21} &= 2\mu\varepsilon_{21} \\ \sigma_{22} &= \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu\varepsilon_{22} \\ \sigma_{23} &= 2\mu\varepsilon_{23} \\ \sigma_{31} &= 2\mu\varepsilon_{31} \\ \sigma_{32} &= 2\mu\varepsilon_{32} \\ \sigma_{33} &= \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu\varepsilon_{33}, \end{aligned} \quad (2.1.7)$$

which can be written as

$$\sigma_{ij} = \lambda(\varepsilon_{kk})\delta_{ij} + 2\mu\varepsilon_{ij}, \quad (2.1.8)$$

where it is assumed that all terms $i, j = 1, 2, 3$ are explicitly considered.