

Bulk modulus κ : The ratio of hydrostatic pressure to the resulting volume change, a measure of the incompressibility of the material. It can be expressed as

$$\kappa = \lambda + \frac{2}{3}\mu. \quad (2.26)$$

Poisson's ratio ν : The ratio of the lateral contraction of a cylinder (being pulled on its ends) to its longitudinal extension. It can be expressed as

$$\nu = \frac{\lambda}{2(\lambda + \mu)}. \quad (2.27)$$

In seismology, we are mostly concerned with the compressional (P) and shear (S) velocities. As we will show later, these can be computed from the elastic constants and the density, ρ :

P velocity, α , can be expressed as

$$\alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}}. \quad (2.28)$$

S velocity, β , can be expressed as

$$\beta = \sqrt{\frac{\mu}{\rho}}. \quad (2.29)$$

Poisson's ratio ν is often used as a measure of the relative size of the P and S velocities; it can be shown that

$$\nu = \frac{\alpha^2 - 2\beta^2}{2(\alpha^2 - \beta^2)}. \quad (2.30)$$

Note that ν is dimensionless and varies between 0 and 0.5 with the upper limit representing a fluid ($\mu = 0$). For a *Poisson solid*, $\lambda = \mu$, $\sigma = 0.25$, and $\alpha/\beta = \sqrt{3}$. Most crustal rocks have Poisson's ratios between 0.25 and 0.30.

2.3.1 Units for Elastic Moduli

The Lamé parameters, Young's modulus, and the bulk modulus all have the same units as stress (i.e., pascals). Recall that

$$1 \text{ Pa} = 1 \text{ N m}^{-2} = 1 \text{ kg m}^{-1} \text{ s}^{-2}.$$

Note that when this is divided by density (kg m^{-3}) the result is units of velocity squared (appropriate for Equations 2.28 and 2.29).